

Problem 2

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sqrt{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{4}}{3} + \frac{\sqrt{5}}{4} + \frac{\sqrt{6}}{5} + \dots$$

Solution

The series can be written as

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n}.$$

Take the limit of the summand as $n \rightarrow \infty$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\sqrt{n+1}}{n} &= \lim_{n \rightarrow \infty} \sqrt{\frac{1}{n^2}(n+1)} \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{1}{n} + \frac{1}{n^2}} \\ &= \sqrt{0+0} \\ &= 0 \end{aligned}$$

Since it's zero, no conclusion can be drawn. Further testing is needed.